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forest management

A Gingrich-Style Stocking Chart for Longleaf Pine (*Pinus palustris* Mill.) Forests

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Because of the dramatic decline in longleaf pine (*Pinus palustris* Mill.) acreage, concern about restoration and management of these ecosystems has increased in recent years and created a need for effective silvicultural management tools. Stocking charts are useful quantitative tools to allocate tree area to meet specific silvicultural objectives including restoration; however, there has not been one created specifically for longleaf pine forests. Because successful management of longleaf pine is often associated with density management at or near the onset of full site occupancy, which is readily determined on a stocking chart, the development of the chart for the species was needed. We developed a Gingrich-style stocking chart for longleaf pine forests using published approaches and models from the literature. Average maximum density (A-line stocking) was determined using forest inventory data whereas onset of full site occupancy (B-line stocking) was derived from an existing open-growth crown width equation. Reduced major axis regression was used to determine size-density relationships because it gives less biased and more efficient estimates than ordinary least squares regression. Previous studies, physiological data, and longleaf pine silvical traits all support the size-density characteristics depicted on this stocking chart. We found that percent stocking was better than basal area as a predictor of tree growth, although the difference between the two measures was not significant in understocked stands. The difference between percent stocking and stand density index as a predictor of tree growth was not statistically significant. With the stocking chart presented in this article, tree area relationships can be effectively and easily used to achieve specific silvicultural objectives.

Keywords: A-line stocking, B-line stocking, basal area, relative density

Tree area allocation during restoration and management of longleaf pine (*Pinus palustris* Mill.) forests is crucial for several reasons. Cone production decreases with increasing stand density (Croker and Boyer 1975); at low basal area (BA; i.e., BA between 6.9 and 9.2 m^2 ha^{-1}), a higher number of cones are generally obtained (Boyer 1990). During the seedling stage, longleaf pine grows very slowly under an overstory canopy and growth rates decrease with increasing overstory density (Boyer 1993). Denser canopies not only reduce seedling growth through competition for light (Brockway et al. 2006) but may also decrease seedling survival through their indirect effects on fire severity. Longleaf pine developed adaptations to survive in an ecosystem that has been subjected to frequent fires ignited by lightning strikes (Landers 1991). Frequent prescribed fire is a common tool used to remove the litter layer that is detrimental to germination of longleaf pine seeds (Boyer 1990) and to reduce competition with hardwoods. In denser stands, more pine needles accumulate on the ground, resulting in a hotter and higher intensity fire and, consequently, higher mortality rates among understory seedlings (Croker and Boyer 1975, Grace and Platt 1995). Therefore, deciding on the adequate posttreatment residual stand density for successful regeneration, recruitment, and growth of longleaf pine is vital. Increasing interest in longleaf pine restoration and management using the optimal stand density requires scientifically based silvicultural management tools for this species.

Stand size-density relationships affect the degree of tree crowding within a stand (Ernst and Knapp 1985) and help influence the intensity of competition among trees for growth resources, including light, water, and nutrients. With an understanding of size-density relationships, forest managers aim to influence tree size, growth, and mortality by altering available tree area (Lhotka and Loewenstein 2008, Puettmann et al. 1993). However, determining the optimal stand densities to achieve specific objectives has been a complex process for forest managers. BA alone is a commonly used density measure when allocating tree area (Zeide 2010). However, available tree area in a stand, at a given BA, varies with average tree diameter (Goelz 1995, Martin 1996). Thus, relative density measures and associated graphical tools such as Gingrich's (1967) stocking chart can offer greater precision than BA alone when allocating tree area through silvicultural manipulation of a forest stand. A Gingrich- (1967) style stocking chart, hereafter referred to as simply a Gingrich stocking chart, shows the average maximum density

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(A-line) that helps to quantify size-density relationships under levels of competition present in normal stand conditions (Gingrich 1967, Larsen et al. 2010). Normal stands are defined as those for which density fluctuates around an equilibrium level at which additional growth is balanced by mortality elsewhere in the stand (Zeide 2004, 2010). The chart also shows the onset of full site occupancy (B-line) that represents the maximum tree area that a tree of a given diameter can occupy under open-grown conditions (Larsen et al. 2010). In addition to stocking charts, density management diagrams (DMDs) graphically depict relationships among yield, density, and mortality at all stages of stand development (Jack and Long 1996, Newton 1997). A DMD has been constructed for longleaf pine (Shaw and Long 2007), but there has not been a Gingrich stocking chart created specifically for longleaf pine forests.

The DMD developed for longleaf pine by Shaw and Long (2007) defines relationships among average tree diameter, tree height, SDI, and stand volume, and it helps quantify maximum size-density and self-thinning relationships (Drew and Flewelling 1979). However, unlike a Gingrich stocking chart, DMDs generally do not include an empirically based estimate of the onset of full site occupancy or canopy closure (i.e., B-line) (Johnson et al. 2009, p. 272). The B-line reference common to stocking charts not only informs density management of an existing stand, but it is also used to determine percent stocking (% stocking) levels that yield the long-term canopy openness necessary for species regeneration and recruitment. Given that some of the most important longleaf pine management issues outlined above relate to regeneration and recruitment under silvicultural systems that maintain partial overstory canopies, the creation of a Gingrich stocking chart for longleaf pine would aid in the development of silvicultural methods for the region's diverse management and forest structure objectives. The existence of a longleaf pine stocking chart would not only complement the Shaw and Long (2007) DMD in terms of informing size-density patterns, but it would also better aid in defining the onset of full site occupancy. In this study, our objective was to develop a Gingrich stocking chart for longleaf pine forests. Specific objectives were to describe the onset of full site occupancy, which is important in the context of managing stand growth, regeneration, and recruitment of longleaf pine, and to compare whether % stocking, stand density index (SDI), or an absolute measure of stand density (BA) was a better predictor of tree growth in longleaf pine stands.

Stocking Chart Composition

Size-density relationships are a component of self-thinning that is a fundamental process of stand development (Zeide 2010). These relationships have been used to develop stand management diagrams (Drew and Flewelling 1977, Gingrich 1967). A tree's "tree area" is usually estimated as the circular area of its crown (Chisman and Schumacher 1940, Pretzsch et al. 2015). The maximum tree area that a tree can occupy is attained when the tree is open-grown and free from competition. Open-grown trees develop the largest crown possible relative to their dbh (Krajicek et al. 1961), and this species-specific relationship is used to define the onset of full site occupancy (B-line). At the other end of the spectrum, the minimum tree area that is necessary for good physiological functions (Johnson et al. 2009, p. 262) is estimated from normal stands (Drew and Flewelling 1977, Gingrich 1967) and used to define the average maximum density (A-line). It has been shown that age and site quality have insignificant influence on tree area relating to average maximum density (Chisman and Schumacher 1940, Williams 1996).

Stand density can be described with absolute or relative measures. Basal area per acre (BA), trees per acre (TPA), and volume per acre are quantitative and absolute measures of stand density. These measures are not comparable across stands because two stands with same TPA or BA may not necessarily have the same level of available tree area if average tree size differs. On the other hand, relative density measures such as % stocking refer to the crowding of trees in relation to a given reference level guided by management objectives (Ernst and Knapp 1985) and are comparable in terms of available tree area across stands. A Gingrich stocking chart combines measures of both absolute and relative density into one graphical tool. Stocking charts illustrate relationships among BA, TPA, and quadratic mean diameter (QMD); % stocking of a stand is obtained based on any two of these three measurements.

Materials and Methods

To develop a Gingrich stocking chart, two key reference curves must be defined: onset of full site occupancy (B-line) and the average maximum density (A-line) (Larsen et al. 2010).

Onset of Full Site Occupancy (B-Line Stocking)

To define the B-line reference curve representing the minimum number of TPA for onset of full site occupancy (N_{OFO}), we used concepts presented by Krajicek et al. (1961) and a linear regression between maximum crown width (CW_{max}) and dbh of open-grown longleaf pine trees developed by Smith et al. (1992) (Equation 1).

$$CW_{max} = b_0 + b_1(QMD) \quad (1)$$

where b_0 and b_1 are coefficients and QMD is the 1-in. diameter class that was used when calculating CW_{max}. Using Equation 1, CW_{max} was calculated in feet for each 1-in. dbh class. Next, for each CW_{max} calculated, N_{OFO} was determined using Equation 2 (Lhotka and Loewenstein 2008). BA (ft^2 ac^{-1}) for each N_{OFO} was calculated using Equation 3.

$$N_{OFO} = \frac{43560}{CW_{max}^{\;2}(\pi/4)} \quad (2)$$

Management and Policy Implications

The fate of longleaf pine forests exploited throughout history in the United States is particularly in the hands of forest managers and landowners. In practice, forest managers commonly use absolute density measures, especially stand basal area, when prescribing residual stand density. However, understanding the inefficiency of absolute density measures in predicting available tree area may encourage forest managers to seek alternative silvicultural tools for effective forest management. Management efforts can be improved by emphasizing the role of relative density measures such as percent stocking when allocating tree area during restoration and management of the existing longleaf pine forests. A graphical tool for relating stand density to tree size was developed in the form of a Gingrich-style stocking chart applicable to longleaf pine forests. The longleaf pine stocking chart outlined in this article can be an important silvicultural management tool for forestry practitioners for informing density management decisions in the ecologically and economically important longleaf pine ecosystems.

Table 1. Descriptive statistics of data sets used for fitting B-Line and A-Line stocking and comparing tocking (%), SDI, and BA (ft^2 ac^{-1}) for tree growth.

Use of data set	Source of data set	Variables	Min	Max	Mean	SD
B-line stocking (n = 81 trees)	Smith et al. (1992)	dbh (in.)	0.9	25.9	9.1	10.4
		Age	1	75	17.1	15.4
		Crown width (ft)	1.97	53.8	19.68	14.43
A-line stocking (n = 26 plots)	USDA Forest Service 2011	BA	141.8	236.9	185.2	27.39
		TPA	59.9	505.9	309.3	108.9
		QMD (in.)	5.51	16.5	10.8	2.4
Comparison of stocking and BA above B-line (n = 134 plots)	USDA Forest Service's Laboratory at Pineville, LA	BA	75.7	193.0	121.6	25.8
		Stocking (%)	40.0	102.0	65.4	12.9
		SDI	41	325	210	42.2
		Age	25.0	65.0	52.0	8.53
		dbh 1 (in.)	5.5	16.5	12	1.94
		dbh 2 (in.)	6.2	17.3	12.6	1.96

dbh 2 refers to the second measurement taken 5 yr after dbh 1.

$$BA = QMD^2(0.005454)\,N_{OFO} \quad (3)$$

Average Maximum Density (A-Line Stocking)

To fit A-line stocking that represents the average maximum density (N_{AMax}) for fully occupied stands, the US Forest Service's Forest Inventory and Analysis (FIA) database for years 2000–2010 was used. Data from plots in the states of Alabama, Georgia, Florida, South Carolina, and Mississippi were downloaded from the FIA website (USDA Forest Service 2011). Only fixed-radius plots were used. Each plot consists of four 24-ft (7.3-m) radius subplots in which all trees with a dbh of 5 in. (12.7 cm) and greater were measured (O'Connell et al. 2014). This study was constrained to single-condition FIA plots; pure even-aged longleaf pine plots in which longleaf pine BA was 90% or more of total plot BA were selected in the database. Data from plots thinned during the inventory period (i.e., 2000–2010) were excluded from the analysis.

To develop an equation that represents the A-line reference (i.e., 100% stocking), the most fully stocked plots from the FIA database were chosen using the following approach. First, SDI was calculated for each plot using Reineke's formula (Reineke 1933; Equation 4).

$$SDI = TPA(QMD/10)^{1.605} \quad (4)$$

Relative stand density (RSD), which is defined as the ratio of actual SDI to the maximum SDI (Solomon and Zhang 2002), was calculated for each plot. Although it has been stated that self-thinning and density-related mortality should begin in stands with an RSD greater than 0.6 (Long 1985, Shaw and Long 2007), we selected plots with relatively higher RSD (>0.7), as suggested by Solomon and Zhang (2002), to ensure that the plots were within the process of self-thinning and fully stocked. Twenty-six plots were identified as having an RSD greater than 0.7, and these plots were used to develop the N_{AMax} equation for the longleaf pine stocking chart (Table 1) . The sample size used to generate average maximum density equations in related studies for other species varies from 9 to 115, with an average of 26 plots across all studies (Comeau et al. 2010, Larsen et al. 2010, Pretzsch and Biber 2005, Solomon and Zhang 2002). Given the average number of plots used for these other tree species, our sample size of 26 plots appears acceptable.

Reineke (1933) suggests that the relationship between the number of trees per unit area (N) and QMD is linear on a log-log scale (Equation 5).

$$\log(N_{AMax}) = b_0 + b_1[\log(QMD)] \quad (5)$$

where b_0 and b_1 are coefficients. The b_0 and b_1 regression coefficients were estimated using data from the 26 FIA plots that had an RSD greater than 0.7. To determine the size-density relationship using Equation 5, both ordinary least squares (OLS) regression and reduced major axis (RMA) regression have been used (Comeau et al. 2010, Lhotka and Loewenstein 2008, Solomon and Zhang 2002, VanderSchaaf and Burkhart 2007). Solomon and Zhang (2002) suggested that RMA regression gives less biased and more efficient estimates than OLS when fitting size-density relationships. The assumptions underlying the statistical procedures affect the results of analyses (Harper 2014). OLS regression assumes that the independent variable is measured without error whereas RMA is based on the assumption that there are errors in the independent variable (Smith 2009). Thus, the slope of the OLS line would be biased with the presence of error in the independent variable (Smith 2009). Because both our independent (i.e., dbh) and dependent (i.e., TPA) variables were subject to error, RMA was more appropriate to fit the line in size-density data. In addition, whereas RMA minimizes the error using both vertical and horizontal distances of data points from the regression line, OLS only uses the vertical distances resulting in more biased estimates (Leduc 1987, Smith 2009). Moreover, OLS does not allow prediction of the independent variable from the dependent variable whereas reciprocal predictions can be done using RMA (Harper 2014, Smith 2009). This also makes RMA a more appropriate regression approach than OLS in density-size relationships.

The slope (β_{RMA}) and intercept (α_{RMA}) of the RMA regression were calculated using Equations 6 and 7 after Solomon and Zhang (2002).

$$\beta_{RMA} = \beta_{OLS}/|Corr_{x,y}| \quad (6)$$

$$\alpha_{RMA} = \mu N - \beta_{RMA}(\mu QMD) \quad (7)$$

where β_{OLS} is the slope of OLS, $Corr_{x,y}$ is the Pearson correlation between number of trees and QMD, μN is the mean tree density, and μQMD is the mean QMD. N_{AMax} for each 1-in. diameter class was calculated using Equation 5. BA for each diameter class was then calculated using Equation 8.

$$BA = QMD^2(0.005454)\,N_{AMax} \quad (8)$$

Finally, B-line and A-line stocking levels were drawn on the same chart using the N_{AMax} and N_{OFO} for average stand diameters between 6 and 20 in. (between 15 and 55 cm) (Lhotka and Loewenstein 2008). Stocking levels below the A-line were determined as a proportion of average maximum density, using *TPA* and *BA* as variables with the same *QMD*.

Comparison of % Stocking, BA, and SDI as a Predictor of Tree Growth

To compare the three measures of stand density (% stocking, BA, and SDI) in their ability to predict diameter growth, long-term data from the US Forest Service's Laboratory at Pineville, Louisiana was used (Goelz and Leduc 2001). The permanent plots were from a combination of seven studies that explored the effects of spacing and thinning on the growth and development of longleaf pine plantations (Goelz and Leduc 2001). The plots were distributed across Alabama, Florida, Louisiana, Mississippi, and Texas, representing most of longleaf pine's current range. Study plots were regularly remeasured for more than 40 years at 5-year intervals (Gonzalez-Benecke et al. 2012). The plots were rectangular, ranging from 0.1 to 0.25 ac in size (Lohrey and Bailey 1977). Plantations are located on both old field and cutover sites (Gonzalez-Benecke et al. 2012). Silt loams were the primary soil texture of the plots. One hundred thirty-four permanent longleaf pine plots from six different sites were chosen from the data set (Table 1). There was an average of 165 trees ac^{-1} (408 ha^{-1}) across all plots, ranging from 60 to 505 trees ac^{-1} (148–1,247 trees ha^{-1}). It should be noted that all study plots for the comparison of % stocking, BA, and SDI had a % stocking that exceeded the B-line reference. Because trees have a maximum tree area below the B-line stocking, individual-tree diameter growth is likely insensitive among varying measures of density or relative density when stands are below the B-line. Thus, to demonstrate that % stocking and BA under B-line are interchangeable, 35 plots that were below B-line (understocked) were also selected from the Pineville data set, and the influence of % stocking and BA on tree growth under B-line was compared. The number of trees ranged from 36 to 330 trees ac^{-1} (89–815 trees ha^{-1}) across all 35 understocked plots.

BA, TPA, and QMD were determined for the selected plots from the Pineville data set. Mean individual-tree diameter growth was calculated for 5-year measurement periods in each plot where no thinning occurred. % stocking of each plot from the Pineville data set was determined using the stocking chart created. For comparative purposes, the SDI of each plot was calculated using Equation 4. The three density measures (% stocking, BA, and SDI) were compared as predictors of diameter growth based on their standardized regression coefficient estimates (βs); a larger absolute value of β means a larger effect (Bring 1994). A mixed-effect model that incorporated BA, % stocking, or SDI as a fixed effect and plots nested within study sites as random effects was used to determine the coefficient values (Hox 2002, p. 11). Standardization facilitates interpretation when comparing effects of different variables with different units within one sample (Hox 2002, p. 21). Standardization was completed using the "standardize" function in the R programming language (R Development Core Team 2010), and the coefficients were standardized. These standardized coefficient estimates represent the increase in the standard deviation of the dependent variable with a 1-unit increase in the standard deviation of the independent variable (Karels et al. 2008) whereas unstandardized coefficients represent the change in the dependent variable when the independent variable is changed by 1 unit (Bring 1994). The comparison is between the changes in standard deviations; the larger the β, the more the independent variable contributes to the prediction of the dependent variable (Bring 1994).

To evaluate statistical significance among the density measures, a bootstrapping methodology was used. Bootstrapping is a statistical technique of resampling with replacement (Singh and Xie 2010). In this method, one sample from the original data set (i.e., 134 samples) was drawn with replacement until a data set of 134 samples was obtained, the mixed-effect model was rerun with the new data set, and the new standardized coefficient estimate (β) was calculated (Steury 2003). This resampling was repeated 1,000 times for each relationship (i.e., diameter growth with BA, diameter growth with % stocking, and diameter growth with SDI), and the confidence interval (CI) of the distribution of the differences in βs of BA, % stocking, and SDI were calculated using the "boot" function in the R programming language (R Development Core Team 2010, Steury 2003). If zero was not contained in the intervals, then it was concluded that the difference between the two variables was statistically significant (Steury 2003). The bootstrapping methods outlined above were also used to compare the ability of BA and % stocking to predict individual-tree, 5-year dbh growth within the plots for which stocking was below the B-line reference level.

Table 2. Parameter estimates and fit statistic of A-Line stocking ($\log(N_{Amax}) = b_0 + b_1[\log(QMD)]$) and crown width-dbh relationships($CW_{max} = b_0 + b_1(dbh)$).

Stocking level	Source of data	Slope	Intercept	R^2
A-line	USDA Forest Service 2011	–1.754	5.377	0.85
B-line	Smith et al. (1992)	0.259	0.113	0.96

Results

Longleaf Pine Stocking Chart

Table 2 gives the slope and intercept for the N_{AMax} and N_{OFO}. The slope of our A-line equation (−1.6244), fit with OLS regression, was close to Reineke's (1933) universal slope of −1.605. On the basis of the RMA regression, the slope of our A-line on a log-log scale was −1.754, steeper than Reineke's slope (Table 2). The maximum SDI for a single plot from the FIA data set was similar to Reineke's (1933) and Shaw and Long's (2007) maximum SDI of 400 for longleaf pine.

Gingrich stocking charts for longleaf pine are presented in Figure 1 (English units) and in Figure 2 (metric units), and data points are shown in Figure 3. The fitted A-line on the chart spans the range between QMD of 6 and 20 in. (15 and 55 cm). It should be noted that the 26 fully stocked plots are in the range of QMD of 7.95–15 in. (20–38 cm). The A-line on our stocking chart ranges between 160 and 218 $ft^2\ ac^{-1}$ (36 and 50 $m^2\ ha^{-1}$) of BA. The B-line on our chart appears nearly flat, ranging between 61.4 and 63.8 $ft^2\ ac^{-1}$ (14.1 and 14.7 $m^2\ ha^{-1}$) of BA. Across a QMD from 6 to 20 in. (from 15 to 55 cm) dbh, the minimum density at which canopy closure occurs varies by less than 3 $ft^2\ ac^{-1}$ (by 4%) in BA whereas % stocking varies by nearly 10% (Figure 1).

Comparison of % Stocking, BA, and SDI as a Predictor of Tree Growth

Study plots from the Pineville data set were well distributed across the range of QMD, BA, and % stocking on the stocking chart (Figure 3). The influence of % stocking ($\beta = 0.69$) on the diameter growth of longleaf pine trees was higher than the influence of BA ($\beta = 0.63$), and the difference between the influences of BA and % stocking on diameter growth was statistically significantly different (CI = 0.025, 0.072). For the fully stocked plots from the Pineville data set, SDI ranged from 125 to 325 across all plots. The difference

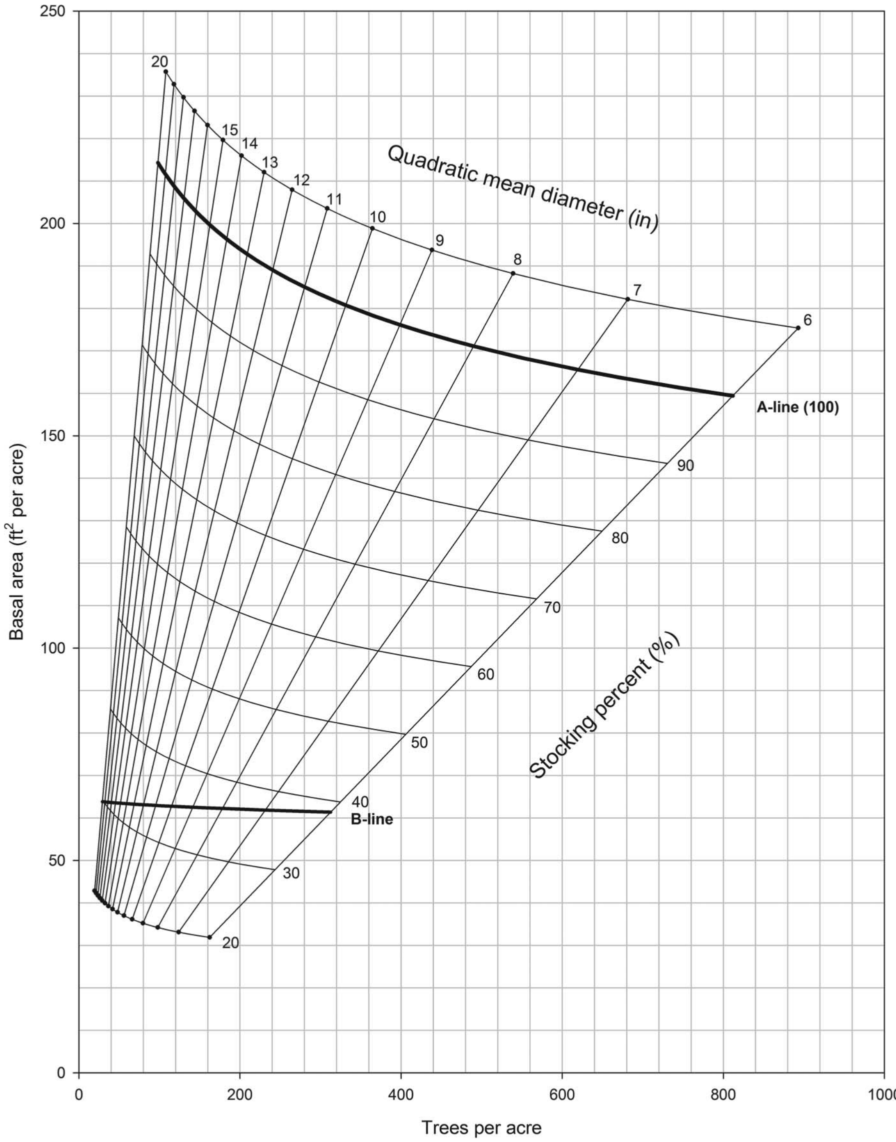

Figure 1. Gingrich-style stocking chart for longleaf pine in English units.

between the influences of % stocking ($\beta = 0.69$) and SDI ($\beta = 0.66$) on the diameter growth of longleaf pine was not significantly different (CI = -0.0001, 0.061). Following the use of plots that were below the B-line stocking, we found that the influence of % stocking ($\beta = 0.76$) and BA ($\beta = 0.76$) was similar on the diameter growth of longleaf pine trees in understocked stands (CI = -0.056, 0.041). These results confirm the expectations that, as long as the density is below canopy closure, trees have a maximum tree area regardless of the average tree diameter of the stand.

Discussion

Longleaf Pine Stocking Chart

The difference between the slope of our A-line and Reineke's (1933) slope can be associated with the use different approaches. Whereas we used RMA regression when fitting A-line because of the reasons stated above, Reineke's (1933) slope was not derived through regression.

On the longleaf pine stocking chart, curved stocking lines may span a substantial range of BAs, especially as % stocking increases. In addition to the competitive influence of increasing crown size by neighboring trees on mortality, physiologic factors such as age-related processes also affect self-thinning (Zeide 2010). When trees become larger, a larger gap is created by the death of a large tree whereas the ability of neighboring trees to close the gap decreases (White and Harper 1970). Younger fully stocked stands have higher absolute density than mature fully stocked stands (Zeide 2005). Stocking lines have a curved shape that steepens with increasing tree size, suggesting that the rate of change between TPA and BA is not linear. Curved stocking lines also indicate that, in the absence of mortality, trees endure more crowding as diameter increases; less space is required by the large diameter trees to support a unit of BA (Johnson et al. 2009, p. 267).

Stands falling above the A-line are considered overstocked and tend toward the A-line as additional growth and density-dependent mortality occur. Stands falling anywhere within the area between the A-line and B-lines are considered fully stocked, meaning that they are capable of completely using the available tree area (Johnson et al. 2009, p. 267). In understocked stands with density below the B-line, there is a surplus of available growing space, and individual tree growth is generally independent of stand density (Williams 1996).

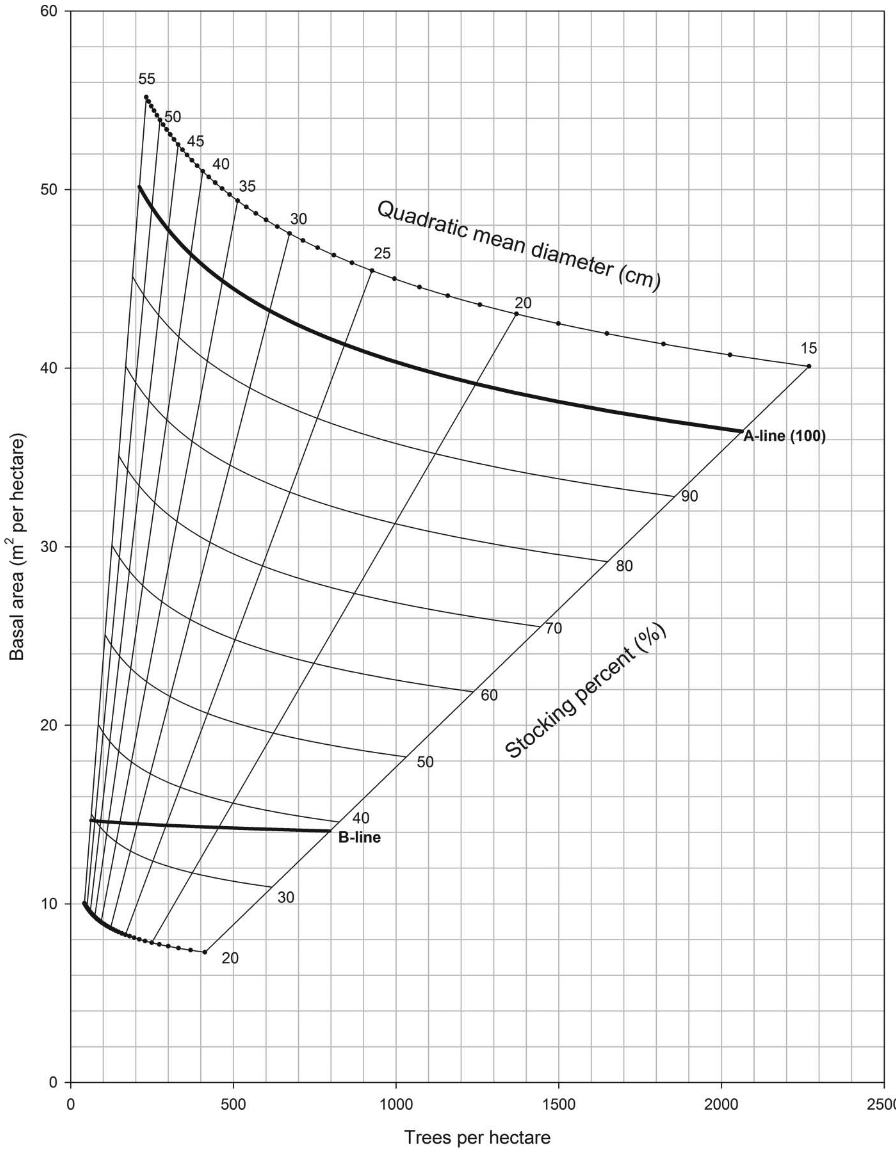

Figure 2. Gingrich-style stocking chart for longleaf pine in metric units.

A-Line Stocking

A-line stocking represents the average maximum density under normally stocked conditions, which refers to undisturbed stands that are at or near maximum density, lacking of gaps, and with relatively uniform spacing (Johnson et al. 2009, p. 260). One significance of the A-line is that, if there is no disturbance, the stocking trend will be toward the A-line whether the current % stocking is above or below the A-line (Gingrich 1967). The high level of BA in the range of the A-line (between 160 and 218 $ft^2\ ac^{-1}$ of BA) suggests that mature longleaf pine trees can survive competition even in dense stands, as stated by Platt et al. (1988). Although longleaf pine seedlings are very intolerant of competition (Brockway and Outcalt 1998), mature and large longleaf trees may become more tolerant of competition because of extensive root systems developed at an early age (Strauss and Ledig 1985); they can better survive competition and continue to grow at a consistent rate after 80–100 years of age when tree area is sufficient (Chapman 1909, The Longleaf Alliance 2016).

To promote individual tree growth, Dickens et al. (2004) indicated that longleaf pine stands should be thinned if BA is greater than 120 $ft^2\ ac^{-1}$ (27.5 $m^2\ ha^{-1}$). BA above this level results in slower individual tree growth and losses to density-related mortality (Kush et al. 2006). This level of BA (120 $ft^2\ ac^{-1}$) ranges from 58 to 75% stocking across the diameter range of the chart. It is interesting to note that an SDI of 250 associated with the onset of self-thinning (Shaw and Long 2007) ranges from 70 to 80% stocking on the stocking chart depending on the average tree diameter. Williams (2003) also noted that density-induced mortality begins at 70% stocking. In the Pineville data set, most mortality occurred in plots with higher than 70% of stocking. It should be noted that these plots were not the same plots that were used to compare % stocking and BA with regard to overstory tree growth. Consistency with the published longleaf pine DMD (Shaw and Long 2007) and the mortality observed in the Pineville plots support the biological relevance and utility of our stocking chart.

B-Line Stocking

Smith et al. (1992) examined the relationship between crown width and dbh for longleaf pine, loblolly pine (*Pinus taeda* L.), and slash pine (*Pinus elliottii* Engelm.) and fitted the regression lines for

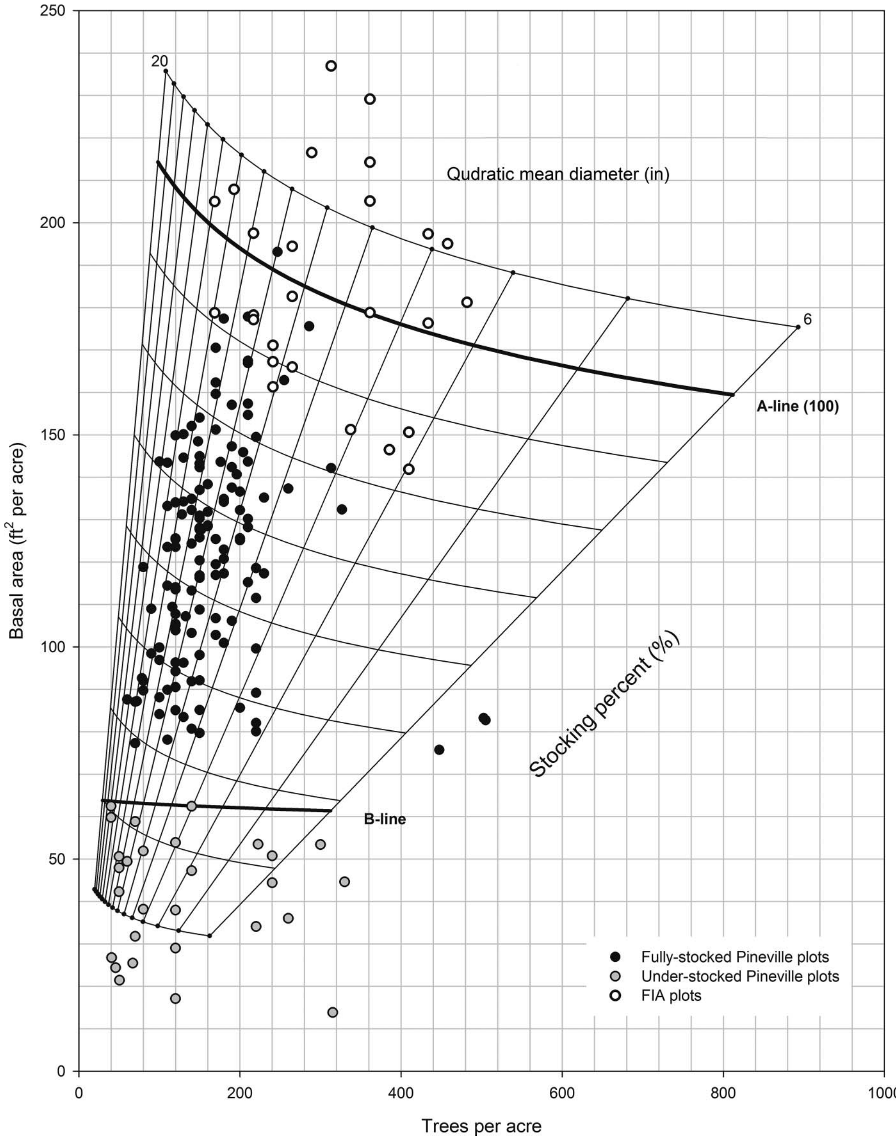

Figure 3. Study plots from FIA and Pineville data set used in this study.

each species. They noted that the relationship for longleaf pine was significantly different than for loblolly and slash pines; the slopes decrease for loblolly and slash pines as dbh increases whereas it decreases at a smaller rate for longleaf pine (Smith et al. 1992). In addition, in other species such as upland central hardwoods (Gingrich 1967), Midwest bottomland hardwoods (Larsen et al. 2010), and southern bottomland hardwoods (Goelz 1995), the B-line closely parallels a given % stocking level. This is in contrast with the slope of the B-line for longleaf pine, which is linear, demonstrating that crown width increases at a constant rate as dbh increases for longleaf pine whereas it increases at a decreasing rate for the other species (Smith et al. 1992). Schwarz (1907, p. 96) also notes this relationship between crown and dbh, observing that crown width increases at a constant rate as diameter increases in open stands of longleaf pine. Thus, in stands with large trees, canopy closure occurs with fewer trees and at a lower BA. This is what makes the B-line nearly flat.

Examining our B-line in relation to assumptions proposed by Shaw and Long (2007) for their longleaf pine DMD, it falls within the range between their lower limit of full site occupancy and the transition from open-grown conditions to a state of competition. Kush et al. (2006) stated that crown closure occurs at approximately 63 $ft^2\ ac^{-1}$ of BA (14.5 $m^2\ ha^{-1}$) in longleaf pine forests. As can be seen on the stocking chart, the B-line ranges from 61.3 to 63.8 $ft^2\ ac^{-1}$ of BA (14.1 to 14.7 $m^2\ ha^{-1}$) across the QMDs. Treatments leading to successful regeneration of longleaf pine stands usually start by decreasing BA to 60 $ft^2\ ac^{-1}$ with a preparatory cut (Croker and Boyer 1975), and this recommended residual density level aligns with the BA range at the onset of full site occupancy (B-line) on the presented stocking chart. In addition, Knapp et al. (2013) observed the influence of canopy structure on the growth of longleaf pine seedlings over three growing seasons and found that rootcollar diameter did not change in stands where BA was higher than 70 $ft^2\ ac^{-1}$ (16 $m^2\ ha^{-1}$), suggesting that densities above B-line stocking are not appropriate for recruitment of longleaf pine seedlings.

Comparison of % Stocking, BA, and SDI

As stated before, two stands with the same BA may occupy different amounts of tree area depending on the average size of the trees. Gingrich (1967) concluded that a stand with larger QMD will occupy less tree area than a stand with smaller QMD for a given BA. Our findings from even-aged longleaf pine stands substantiate this statement, suggesting that tree growth can be explained better by % stocking rather than BA. In addition, RSD such as % stocking has the ability to measure and compare stands that differ in age, tree size, BA, or species composition (Ernst and Knapp 1985).

The onset of full site occupancy is not explicitly shown on a DMD whereas it is empirically defined using the open-grown trees in the stocking chart (Johnson et al. 2009, p. 272). Shaw and Long (2007) suggested an SDI of 100 for the onset of competition because it is generally accepted (Long 1985). In addition to SDI lines as the index of relative density, the longleaf pine DMD has a set of curves representing the gross stand volume (Shaw and Long 2007). Although they possess different attributes and representations of growing space, this stocking chart and the Shaw and Long (2007) DMD each serve as a quantitative aid for density management in longleaf pine. The stocking chart presented here works to enhance the understanding of tree area relationships, especially related to the onset of full site occupancy and its potential implications in regeneration and recruitment.

Conclusions

In practice, foresters working in longleaf pine commonly use BA when prescribing residual stand density. However, the degree of stand stocking is inaccurately predicted by absolute measures such as BA. % stocking may be a better indicator of stand density than BA because the amount of available tree area changes based on the average tree diameter at a given BA, although this may not be the case if stand density is below canopy closure. To overcome the deficiencies of BA as a metric for describing tree area availability, DMDs such as stocking charts show relative density along with the absolute measures (i.e., TPA, dbh, or BA). A Gingrich stocking chart was developed for longleaf pine forests, and we found that % stocking was a better predictor of tree growth than BA. This chart offers greater precision than BA alone when allocating tree area through silvicultural manipulation of a stand.

Existing studies, physiological data, and longleaf pine silvical traits all support the size-density patterns represented in this stocking chart. This alternative representation of tree area relationships presented in our stocking chart provides forestry practitioners with an important tool for informing density management decisions in the ecologically and economically important longleaf pine ecosystems of the southern United States. Some of the most important management issues in longleaf pine forests relate to regeneration and recruitment under silvicultural systems that maintain stands under canopy closure. Thus, the minimum density of canopy closure estimated in the stocking chart helps determine stocking levels that yield the long-term canopy openness necessary for the successful regeneration and recruitment of longleaf pine. Unlike the stocking chart, the longleaf pine DMD predicts stand top height and volume as a function of QMD and TPA, which may be more practical for some specific silvicultural objectives such as wildlife management (Shaw and Long 2007). Thus, this chart does not intend to replace the longleaf pine DMD; rather, it aims to complement the DMD in terms of informing size-density patterns.

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